Solution of Demchik's Model of Water Filtration Using the Method of Separation of Variables

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Abstract

The solution of the model developed by Demchik [2] was obtained using variable seperable method.

Keywords: Separation of variables, water filtration.

1.0 Introduction

Demchik[1] discussed a mathematical model of water filtration at a decreasing rate which generalizes Mint's model [3]. In this paper, the method of separation of variables will be used to solve the model developed by Demchik [2].

2.0 The Model

Consider the following system of model equations with the appropriate initial boundary conditions.

$$U_{xt}(x,t) + b(t)U_t(x,t) - PU(x,t) = 0$$

$$b(t) = \beta {v_0}^{-1}(1+\gamma t), \quad \rho = \beta {v_0}^{-1}\gamma(z-1) \qquad (1.1)$$

$$U(x,0) = C_0 e^{-\beta {v_0}^{-1}}x, \quad U(0,t) = C_0(1+\gamma t)^z, \; z = a_0 v_0 \gamma^{-1}$$

Together with the relation ....

$$C(x,t) = u(1+\gamma t)^{-z}, \quad \frac{\partial \rho\,(x,t)}{\partial t} = \beta C - a(t)p(x,t) \qquad (1.2)$$

$$\mathrm{V(t)} = \frac{v_0}{1+\gamma t}\ , \qquad a(t) = a_0 v(t)$$

Where $v_0, \ \gamma, \ a_0$ are constants. And x is the coordinate along the thickness of the filter, v(t) is the filtration rate, C(x,t) and P(x,t) are the required concentration of impurities suspended in the liquid and sediment respectively, β is the kinetic coefficient and C_0 is the impurity concentration in the liquid at the filter net.

Seeking a solution of the form

$$U(x,t) = X(x)T(t) \qquad (1.3)$$

Where X(x) is a function of x only and T(t) is a function of t only. Differentiating (1.3) with respect to x and t, we have

$$U_x(x,t) = X'(x)T(t)$$

$$U_t(x,t) = X(x)T'(t) \tag{1.4}$$

$$U_{xt}(x,t) = X'(x)T'(t)$$

Hence (1.1) can be written as

$$X'(x)T'(t) + \mathrm{b(t)}\, X(x)T'(t) - \rho X(x)T(t) = 0$$

Giving

$$X'(x)T'(t) + X(x)[\mathrm{b(t)}\, T'(t) - pT(t)] = 0$$

$$\frac{X'(x)}{X(x)} + \frac{\mathrm{b(t)}\, T'(t) - pT(t)}{T'(t)} = 0$$

The equality holds if both sides are equal to a constant say. We have,

$$\frac{X'(x)}{X(x)} = \lambda$$

Implies $\quad X'(x) - \lambda X(x) = 0$

The auxiliary form of the equation is

$$X'(x) = Ae^{\lambda x}$$

Also,

$$\frac{\mathrm{PT(t)} - \mathrm{b(t)}T'(t)}{T'(t)} = \lambda$$

The auxiliary form is given as

$$T(t) = Be^{\frac{P}{b(t)+\lambda}t}$$

Hence

$$U(x,t) = Ae^{\lambda x} \cdot Be^{\frac{P}{b(t)+\lambda}t}$$

$$U(x,t) = Ce^{\lambda x}.e^{\frac{P}{b(t)+\lambda}t}$$

From equation (1.1), we have

$\rho = \beta {v_0}^{-1}\gamma(z-1)$ and $b(t) = \beta {v_0}^{-1}(1+\gamma t)$, $z = a_0 v_0 \gamma^{-1}$

Then,

$$U(x,t) = Ce^{\lambda x}.e^{\frac{(a_{0-v_0^{-1}\gamma})\beta}{\beta {v_0}^{-1}(1+\gamma t)+\lambda}t}$$

$$Ce^{\lambda x+\frac{(a_{0-v_0^{-1}\gamma)\beta}}{\beta v_0{}^{-1}(1+\gamma t)+\lambda}t} \quad (1.5)$$

Applying the condtion U(x,0)=C_0exp(−$\beta v_0{}^{-1}x$), we have

$$C_0 e^{-\beta v_0{}^{-1}x} = Ce^{\lambda x}$$

$$C_0 = Ce^{\lambda x}.e^{\beta v_0{}^{-1}x}$$

$$C_0 = Cexp(\lambda + \beta v_0{}^{-1})x \quad (1.6)$$

Also applying $U(0,t) = C_0(1+\gamma t)^z$

$$C_0(1+\gamma t)^z = Cexp\left[\frac{(a_0 - v_0^{-1}\gamma)\beta}{\beta v_0{}^{-1}(1+\gamma t)+\lambda}t\right] \quad (1.7)$$

From equation (1.5) and (1.6), (1.7) can be written as

$$Cexp(\lambda + \beta v_0{}^{-1})x\,.\,(1+\gamma t)^z = Cexp\left[\frac{(a_0 - v_0^{-1}\gamma)\beta}{\beta v_0{}^{-1}(1+\gamma t)+\lambda}t\right]$$

Taking logarithm to base e of both sides, we have,

$$\log_e exp(\lambda + \beta v_0{}^{-1})x + \log_e(1+\gamma t)^{a_0 V_0 \gamma^{-1}} = \log_e \exp(a_{0-v_0^{-1}\gamma)\beta t} - \log_e \exp(\ \beta v_0{}^{-1}(1+\gamma t)+\lambda)$$

Giving

$$(\lambda + \beta v_0{}^{-1})x + a_0 V_0 \gamma^{-1}\log_e(`1+\gamma t) = (a_0 - v_0^{-1}\gamma)\beta t - (\ \beta v_0{}^{-1}(1+\gamma t)+\lambda)$$

Which gives

$$a_0\beta t - \beta V_0^{-1}(x+2\gamma t+1) = \lambda(x+1) + a_0 V_0 \gamma^{-1}\log_e(1+\gamma t)$$

Hence

$$\beta = \frac{\lambda(x+1) + a_0 V_0 \gamma^{-1}\ln(1+\gamma t)}{a_0 t - V_0^{-1}(x+2\gamma t+1)}$$

For the value of λ, we have

$$\lambda = \frac{a_0(\beta t - V_0\gamma^{-1}\log_e(1+\gamma t)) - -\beta V_0^{-1}(x+2\gamma t+1)}{x+1}$$

3.0 CONCLUSION

It was shown that a unique solution exits for the model developed by Demchik [2], using the separation of variable method. As Augusto F.B et.al [1] was able to show the existence of a unique solution to the filtration problem and also the existence of the optimal control for the problem.

References

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